A NOTE ON METRIC SPACES WITH CONTINUOUS MIDPOINTS∗

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Abstract

A metric space \((X, d)\) is a continuous midpoint space if there is a continuous map \(\mu : X \times X \to X\) such that, for all \((a, b) \in X \times X\),
\[d(a, \mu(a, b)) = \frac{1}{2}d(a, b) = d(b, \mu(a, b)).\]
A closed subset \(C\) of a complete continuous midpoint space is convex if \(\forall (a, b) \in C \times C, \mu(a, b) \in C\). Under suitable, but natural, assumptions continuous midpoint spaces are absolute retracts; Browder, Michael or Cellina like continuous selection theorems hold; bounded closed convex sets have the fixed point property for nonexpansive maps. Hyperconvex metric spaces, Cartan-Hadamard manifolds and more generally Hadamard spaces or metric spaces with non positive curvature in the sense of Busemann are continuous midpoint spaces.

MSC: 47H10, 51D99, 53C70, 54C65

keywords: Selections, Fixed Points, Geodesic convexity, Busemann spaces, cat(0) spaces, Hyperconvex spaces

1 Introduction

Given two points \(a\) and \(b\) of a metric space \((X, d)\) a point \(m\) of \(X\) is a midpoint for the pair \((a, b)\) if \(d(a, m) = \frac{1}{2}d(a, b) = d(b, m)\). For all pairs of points of a complete metric space \((X, d)\) to have a midpoint it is

∗Accepted for publication on 20.09.09
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